Chapter 3
Energy and Exergy Efficiencies

3.1 Introduction

Reductions in energy use can assist in attaining energy security objectives. Also, efficient energy utilization and the introduction of renewable energy technologies can significantly help solve environmental issues. Increased energy efficiency benefits the environment by avoiding energy use and the corresponding resource consumption and pollution generation. From an economic as well as an environmental perspective, improved energy efficiency has great potential [2].

An engineer designing a system is often expected to aim for the highest reasonable technical efficiency at the lowest cost under the prevailing technical, economic, and legal conditions, and with regard to ethical, ecological, and social consequences. Exergy methods can assist in such activities and offer unique insights into possible improvements with special emphasis on environment and sustainability. Exergy analysis is a useful tool for addressing the environmental impact of energy resource utilization, and for furthering the goal of more efficient energy-resource use, for it enables the locations, types, and true magnitudes of losses to be determined. Also, exergy analysis reveals whether and by how much it is possible to design more efficient energy systems by reducing inefficiencies.

Efficiency is a measure of the effectiveness and/or performance of a system. Although it may take different forms, depending on the application and purpose, it can generally be defined as

$$\eta = \frac{\text{Desired output}}{\text{Required input}} \tag{3.1}$$

The definition of energy efficiency is based on the first law of thermodynamics. It is denoted by η. It may take different forms and different names depending on the type of the system. It may be written as

$$\eta = \frac{\text{Energy output}}{\text{Energy input}} = \frac{E_{\text{out}}}{E_{\text{in}}} = 1 - \frac{E_{\text{loss}}}{E_{\text{in}}} \tag{3.2}$$

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where

$$E_{\text{in}} = E_{\text{output}} + E_{\text{loss}} \tag{3.3}$$

or in rate form,

$$\eta = \frac{\dot{E}_{\text{out}}}{\dot{E}_{\text{in}}} = 1 - \frac{\dot{E}_{\text{loss}}}{\dot{E}_{\text{in}}} \tag{3.4}$$

An alternative way of expressing energy efficiency is

$$\eta = \frac{\text{Energy recovered}}{\text{Energy expended}} = \frac{E_{\text{recovered}}}{E_{\text{expended}}} = 1 - \frac{E_{\text{loss}}}{E_{\text{expended}}} \tag{3.5}$$

where

$$E_{\text{expended}} = E_{\text{recovered}} + E_{\text{loss}} \tag{3.6}$$

or in rate form,

$$\eta = \frac{\dot{E}_{\text{recovered}}}{\dot{E}_{\text{expended}}} = 1 - \frac{\dot{E}_{\text{loss}}}{\dot{E}_{\text{expended}}} \tag{3.7}$$

Both (3.2) and (3.5) may be used to find the energy efficiency of a system but one may be more appropriate than the other depending on the system and application. They may turn out to be equivalent in some cases and different in others.

The definition of exergy efficiency is based on the second law of thermodynamics. It is also called second law efficiency or exergetic efficiency. Some sources also call it effectiveness. In this book we use exergy efficiency and second law efficiency interchangeably. Effectiveness is used with a different meaning for the performance of some devices.

Exergy efficiency may take different forms depending on the type of the system. It is denoted by η_{ex}, η_{II}, or ε. In this book, we use ε for the exergy efficiency symbol because it is easier to distinguish from the energy efficiency symbol. Exergy efficiency is generally expressed as

$$\varepsilon = \frac{\text{Exergy output}}{\text{Exergy input}} = \frac{X_{\text{out}}}{X_{\text{in}}} = 1 - \frac{X_{\text{dest}}}{X_{\text{in}}} \tag{3.8}$$

or in rate form,

$$\varepsilon = \frac{\dot{X}_{\text{out}}}{\dot{X}_{\text{in}}} = 1 - \frac{\dot{X}_{\text{dest}}}{\dot{X}_{\text{in}}} \tag{3.9}$$

where

$$\dot{X}_{\mathrm{in}} = \dot{X}_{\mathrm{out}} + \dot{X}_{\mathrm{dest}} \tag{3.10}$$

Conceptually, second law efficiency is a measure of perfection. Thermodynamic perfection is reversibility. The second law dictates that no process can be better than a corresponding reversible process (in everyday terms, nothing can be more perfect than perfect), as that would be a violation of the second law. Therefore, reversible operation is the best possible mode of operation of a device, and thus it is natural that the second law efficiency be 1 or 100% for operations that involve no irreversibilities or imperfections. This sets the upper limit for second law efficiency, and current practice adheres to it.

What is essentially lacking is a fundamental definition for the lower limit of second law efficiency, which should be 0 or 0%. We should establish that the second law efficiency of a device or process that destroys the entire exergy it consumes is zero. The way to accomplish this is to change the general definition of second law efficiency from $\varepsilon =$ (Exergy output)/(Exergy input) to $\varepsilon =$ (Exergy recovered)/(Exergy expended). That is:

$$\varepsilon = \frac{X_{\mathrm{recovered}}}{X_{\mathrm{expended}}} = 1 - \frac{X_{\mathrm{dest}}}{X_{\mathrm{expended}}} \tag{3.11}$$

or in rate form,

$$\varepsilon = \frac{\dot{X}_{\mathrm{recovered}}}{\dot{X}_{\mathrm{expended}}} = 1 - \frac{\dot{X}_{\mathrm{dest}}}{\dot{X}_{\mathrm{expended}}} \tag{3.12}$$

where

$$\dot{X}_{\mathrm{expended}} = \dot{X}_{\mathrm{recovered}} + \dot{X}_{\mathrm{dest}} \tag{3.13}$$

or alternately,

$$\varepsilon = \frac{\dot{X}_{\mathrm{delivered}}}{\dot{X}_{\mathrm{consumed}}} = 1 - \frac{\dot{X}_{\mathrm{dest}}}{\dot{X}_{\mathrm{consumed}}} \tag{3.14}$$

That is, the second-law efficiency of a device is the ratio of the exergy recovered (or delivered) by the device to the exergy expended (or consumed) by the device. This way, the second law efficiency of a device represents its ability to convert one form of exergy into another form (as from thermal to mechanical or vice versa). And, it refers to the resource and it puts the emphasis on the best utilization of a resource.

The difference between the two definitions may appear subtle, but it is fundamental. Here $\dot{X}_{\mathrm{expended}}$ represents the portion of the exergy coming from the resource. It is the shaft work input in the case of a compressor, and the decrease

in the exergy of steam (difference between exergy values at inlet and outlet) in the case of a steam turbine. Exergy recovered is the portion of the expended exergy that is retained as exergy, the portion that is saved from destruction within the system during the process.

Both (3.8) and (3.11) may be used to find the exergy efficiency of a system. In this book, we provide exergy efficiency formulations based on both approaches. However, for the reasons explained above, we recommend using the exergy recovered/exergy expended approach [(3.11)].

It should be mentioned that Cornelissen et al. [13] and Kotas [5] provide an exergy efficiency relation as the ratio of the desired exergy output to the exergy used. They call this rational efficiency. Here, exergy output is all exergy transfer from the system, plus any by-product that is produced by the system, whereas exergy used is the required exergy input for the process to be performed. This is similar to the exergy efficiency definition as the ratio of product to the fuel where the fuel represents the resources expended to generate the product [5, 13].

$$\varepsilon = \frac{\text{Product}}{\text{Fuel}} \tag{3.15}$$

Here, both product and fuel must be expressed in exergy terms.

The second law efficiency relation given by (3.11) is essentially the same as the rational efficiency definition of Cornelissen et al. [13] and Kotas [5]. However, most researchers utilize (3.8) for calculating exergy efficiencies and (3.11) is rarely used.

3.2 Efficiencies of Cyclic Devices

A heat engine is a device that converts heat to work. A steam power plant, a gas-turbine power plant, an internal combustion engine, a solar thermal plant, and a geothermal power plant are some familiar examples. Consider a heat engine as shown in Fig. 3.1. The high-temperature resource at T_H supplies heat to the heat engine at a rate of $\dot{Q}_H$. Some of this heat is converted to work $\dot{W}_{\text{out}}$ and the remaining heat $\dot{Q}_L$ is rejected to a low-temperature medium at T_L. The energy efficiency of this cycle is called thermal efficiency, and is expressed as the ratio of work produced to the heat supplied:

$$\eta_{\text{th}} = \frac{W_{\text{out}}}{Q_H} = 1 - \frac{Q_L}{Q_H} \tag{3.16}$$

Now, we consider a refrigeration or heat pump as shown in Fig. 3.2. A household refrigerator and an air-conditioning system used for cooling and heating are some familiar examples of such devices. Here heat at the rate of $\dot{Q}_L$ is absorbed from the low-temperature reservoir at T_L and heat at the rate of $\dot{Q}_H$ is rejected to the

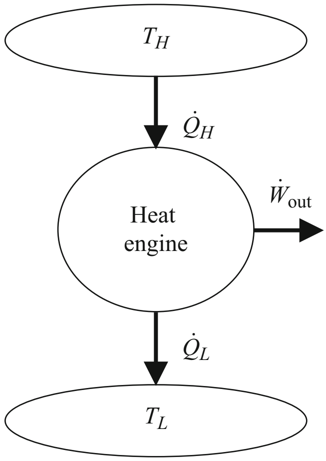


Fig. 3.1 Schematic of a basic heat engine

high-temperature reservoir at T_H. A power input $\dot{W}_{\text{in}}$ is needed for the operation of the cycle. The cycle is called a refrigerator if the purpose is to keep the low-temperature space at T_L and it is called a heat pump if the purpose of the cycle is to keep the high-temperature medium at T_H.

Because the desired output is different for a refrigerator and heat pump, their efficiencies are defined differently. If we utilize the general definition of efficiency, "desired output/required input," in this case, the performance of a refrigerator and a heat pump can be expressed by their coefficient of performance (COP):

$$\text{COP}_{\text{R}} = \frac{\dot{Q}_L}{\dot{W}_{\text{in}}} = \frac{\dot{Q}_L}{\dot{Q}_H - \dot{Q}_L} \tag{3.17}$$

$$\text{COP}_{\text{HP}} = \frac{\dot{Q}_H}{\dot{W}_{\text{in}}} = \frac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L} \tag{3.18}$$

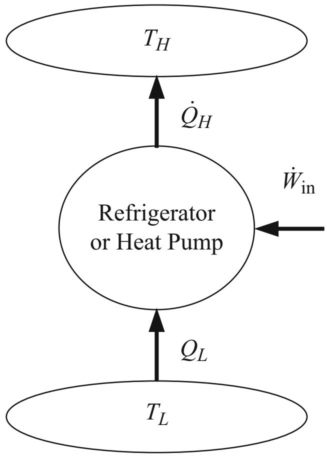
We are careful not to name this as efficiency because the COP values may be lower or greater than unity.

A heat engine that consists of all reversible processes is called a reversible heat engine or a Carnot heat engine. The thermal efficiency of a Carnot heat engine may be expressed by the temperatures of two reservoirs with which the heat engine exchanges heat (Fig. 3.1):

$$\eta_{\text{th, rev}} = 1 - \frac{T_L}{T_H} \tag{3.19}$$

where T_H is the source temperature and T_L is the sink temperature where heat is rejected (i.e., lake, ambient air, etc.). This is the maximum thermal efficiency a heat engine operating between two reservoirs at T_H and T_L can have.

Fig. 3.2 Schematic of a basic refrigeration or heat pump



Because all processes in a Carnot cycle are reversible, the cycle can be reversed. In this case we obtain a reversed Carnot cycle. A refrigerator or heat pump operating on a reversed Carnot cycle (Fig. 3.2) would have the maximum COP values at the given temperature limits T_L and T_H, and they are expressible as

$$\mathrm{COP}_{\mathrm{R,\,rev}} = \frac{T_L}{T_H - T_L} \tag{3.20}$$

$$\mathrm{COP}_{\mathrm{HP,\,rev}} = \frac{T_H}{T_H - T_L} \tag{3.21}$$

Consider two heat engines, both having a thermal efficiency of 30%. One of the engines (engine A) receives heat from a source at 600 K, and the other one (engine B) from a source at 1,000 K. Both engines reject heat to a medium at 300 K. At first glance, both engines seem to be performing equally well. When we take a second look at these engines in light of the second law of thermodynamics, however, we see a totally different picture. These engines, at best, can perform as reversible engines, in which case their efficiencies in terms of the Carnot cycle become

$$\eta_{\mathrm{th,\,rev},\,A} = \left(1 - \frac{T_0}{T_{\mathrm{source}}}\right)_A = 1 - \frac{300\,\mathrm{K}}{600\,\mathrm{K}} = 0.5 \text{ or } 50\%$$
$$\eta_{\mathrm{th,\,rev},\,B} = \left(1 - \frac{T_0}{T_{\mathrm{source}}}\right)_B = 1 - \frac{300\,\mathrm{K}}{1000\,\mathrm{K}} = 0.7 \text{ or } 70\%$$

Engine A has a 50% useful work potential relative to the heat provided to it; engine B has 70%. Now it is becoming apparent that engine B has a greater work potential made available to it and thus should do a lot better than engine A. Therefore, we can say that engine B is performing poorly relative to engine A even though both have the same thermal efficiency.

It is obvious from this example that first law efficiency alone is not a realistic measure of performance of engineering devices. To overcome this deficiency, we define an exergy efficiency (or second law efficiency) for heat engines as the ratio of the actual thermal efficiency to the maximum possible (reversible) thermal efficiency under the same conditions:

$$\varepsilon = \frac{\eta_{\text{th}}}{\eta_{\text{th, rev}}} \tag{3.22}$$

Based on this definition, the exergy efficiencies of the two heat engines discussed above become

$$\varepsilon_A = \frac{0.30}{0.50} = 0.60 \text{ or } 60\%$$
$$\varepsilon_B = \frac{0.30}{0.70} = 0.43 \text{ or } 43\%$$

That is, engine A is converting 60% of the available work potential to useful work. This ratio is only 43% for engine B.

The second-law efficiency can be expressed for work-producing devices such as a turbine as the ratio of the useful work output to the maximum possible (reversible) work output:

$$\varepsilon = \frac{\dot{W}_{\text{out}}}{\dot{W}_{\text{rev, out}}} \tag{3.23}$$

This definition is more general because it can be applied to processes (in turbines, piston–cylinder devices, etc.) as well as to cycles. Note that the exergy efficiency cannot exceed 100%. We can also define an exergy efficiency for work-consuming noncyclic (such as compressors) and cyclic (such as refrigerators) devices as the ratio of the minimum (reversible) work input to the useful work input:

$$\varepsilon = \frac{\dot{W}_{\text{rev, in}}}{\dot{W}_{\text{in}}} \tag{3.24}$$

For cyclic devices such as refrigerators and heat pumps, it can also be expressed in terms of the coefficients of performance as

$$\varepsilon = \frac{\text{COP}}{\text{COP}_{\text{rev}}} \tag{3.25}$$

In the above relations, the reversible work should be determined by using the same initial and final states as in the actual process.

Example 3.1 A geothermal power plant uses geothermal liquid water at 160°C at a rate of 440 kg/s as the heat source, and produces 15 MW of net power in an environment at 25°C. Determine the thermal efficiency, the exergy efficiency, and the total rate of exergy destroyed in this power plant.

Solution The properties of geothermal water at the inlet of the plant and at the dead-state are obtained from steam tables to be

$$\begin{aligned}
&T_1 = 160°\mathrm{C},\ \text{liquid} \longrightarrow h_1 = 675.47\,\mathrm{kJ/kg}, s_1 = 1.9426\,\mathrm{kJ/kg.K}\\
&T_0 = 25°\mathrm{C},\ P_0 = 1\,\mathrm{atm} \longrightarrow h_0 = 104.83\,\mathrm{kJ/kg}, s_0 = 0.36723\,\mathrm{kJ/kg.K}
\end{aligned}$$

The energy of geothermal water may be taken to be the maximum heat that can be extracted from the geothermal water, and this may be expressed as the enthalpy difference between the state of geothermal water and the dead-state:

$$\dot{E}_{\mathrm{in}} = \dot{m}(h_1 - h_0) = (440\,\mathrm{kg/s})[(675.47 - 104.83)\,\mathrm{kJ/kg}] = 251,080\,\mathrm{kW}$$

The exergy of geothermal water is

$$\begin{aligned}
\dot{X}_{\mathrm{in}} &= \dot{m}[(h_1 - h_0) - T_0(s_1 - s_0)]\\
&= (440\,\mathrm{kg/s})[(675.47 - 104.83)\,\mathrm{kJ/kg} + 0\\
&\quad - (25 + 273\,\mathrm{K})(1.9426 - 0.36723)\,\mathrm{kJ/kg.K}] = 44,525\,\mathrm{kW}
\end{aligned}$$

The thermal efficiency of the power plant is

$$\eta_{\mathrm{th}} = \frac{\dot{W}_{\text{net, out}}}{\dot{E}_{\mathrm{in}}} = \frac{15,000\,\mathrm{kW}}{251,080\,\mathrm{kW}} = 0.0597 \text{ or } 6.0\%$$

The exergy efficiency of the plant is the ratio of power produced to the exergy input to the plant:

$$\varepsilon = \frac{\dot{W}_{\text{net, out}}}{\dot{X}_{\mathrm{in}}} = \frac{15,000\,\mathrm{kW}}{44,525\,\mathrm{kW}} = 0.337 \text{ or } 33.7\%$$

The exergy destroyed in this power plant is determined from an exergy balance on the entire power plant to be

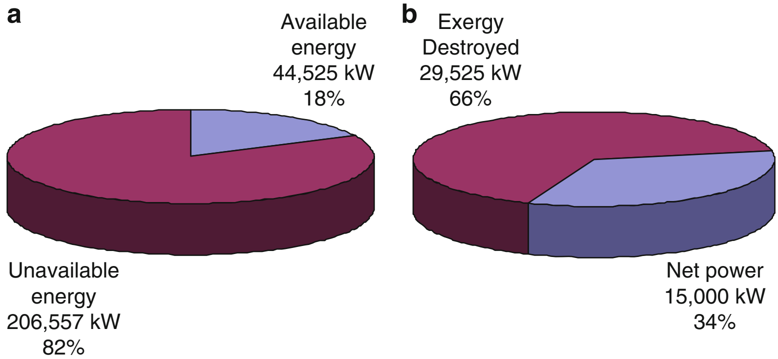


Fig. 3.3 (**a**) Only 18% of the energy of geothermal water is available for converting to power. (**b**) Only 34% of the exergy of geothermal water is converted to power and the remaining 66% is lost

$$\dot{X}_{\text{in}} - \dot{W}_{\text{net, out}} - \dot{X}_{\text{dest}} = 0$$
$$44,525 - 15,000 - \dot{X}_{\text{dest}} = 0 \longrightarrow \dot{X}_{\text{dest}} = 29,525\,\text{kW}$$

The results of this example are illustrated in Fig. 3.3a, b. The exergy of geothermal water constitutes only 18% of its energy, due to its well temperature. The remaining 82% is not available for useful work and it cannot be converted to power even by a reversible heat engine. Only 34% of exergy entering the plant is converted to power and the remaining 66% is lost. In geothermal power plants, the used geothermal water typically leaves the power plant at a temperature much greater than the environment temperature and this water is reinjected back to the ground. The total exergy destroyed (29,525 kW) includes the exergy of this reinjected brine.

In a typical binary-type geothermal power plant, geothermal water would be reinjected back to the ground at about 90°C. This water can be used in a district heating system. Assuming that geothermal water leaves the district at 70°C with a drop of 20°C during the heat supply, the rate of heat that could be used in the district system would be

$$\dot{Q}_{\text{heat}} = \dot{m}c\Delta T = (440\,\text{kg/s})(4.18\,\text{kJ/kg}\cdot^{\circ}\text{C})(20^{\circ}\text{C}) = 36,780\,\text{kW}$$

where c is the specific heat of water. This 36,780 kW heating is in addition to the 15,000 kW power generated. The energy efficiency of this cogeneration system would be (15,000 + 36,780)/251,080 = 0.206 or 20.6%. The energy efficiency increases from 6.0% to 20.6% as a result of incorporating a district heating system into the power plant.

The exergy of heat supplied to the district system is simply the heat supplied times the Carnot efficiency, which is determined as

$$\dot{X}_{\text{heat}} = \dot{Q}_{\text{heat}}\left(1 - \frac{T_0}{T_{\text{source}}}\right) = (36{,}780\ \text{kW})\left(1 - \frac{278\ \text{K}}{353\ \text{K}}\right) = 7814\ \text{kW}$$

where the source temperature is the average temperature of geothermal water (80°C = 353 K) when supplying heat, and the environment temperature is taken as 5°C (278 K). This corresponds to 26.5% (7,814/29,525 = 0.265) of the exergy destruction.

3.3 Efficiencies of Steady-Flow Devices

In this section we provide various efficiencies used for steady-flow devices such as turbines, compressors, pumps, nozzles, diffusers, and heat exchangers. A power plant or a refrigeration system consists of a number of these steady-flow devices, and improving the performance of these devices would improve the performance of the entire plant or system.

3.3.1 Turbine

A fluid is expanded in a turbine to produce power. Steam and gas turbines are considered here. Turbines are normally well insulated so that their operation can be assumed to be adiabatic. The performance of an adiabatic turbine is usually expressed by isentropic (adiabatic) efficiency.

Consider a turbine with inlet state 1 with temperature T_1 and pressure P_1 and an exit state 2 with temperature T_2 (or steam quality) and pressure P_2. The power output from this compressor would be maximum if the fluid were expanded reversibly and adiabatically (i.e., isentropically) between the given initial state and given exit pressure. The isentropic efficiency is then the ratio of actual power to the isentropic power:

$$\eta_{isen,\ \text{turbine}} = \frac{\dot{W}_{\text{actual}}}{\dot{W}_{\text{isentropic}}} = \frac{\dot{m}(h_1 - h_2)}{\dot{m}(h_1 - h_{2s})} \tag{3.26}$$

where $\dot{m}$ is the mass flow rate of fluid (kg/s) and h_s is the enthalpy of the fluid at the turbine outlet if the process were isentropic. This enthalpy may be obtained from exit pressure and exit entropy (equal to inlet entropy). Kinetic and potential energy changes are neglected.

Exergy efficiency of an adiabatic turbine may be determined from an "exergy recovered/exergy expended" approach. In this case, the exergy resource is steam, and exergy expended is the exergy supplied to steam to turbine, which is the decrease in the exergy of steam as it passes through the turbine. Exergy recovered is the shaft work. Taking state 1 as the inlet and state 2 as the outlet, the second law efficiency is expressed as

$$\varepsilon_{\text{turbine}-1} = \frac{\dot{X}_{\text{recovered}}}{\dot{X}_{\text{expended}}} = \frac{\dot{W}_{\text{out}}}{\dot{X}_1 - \dot{X}_2} = \frac{\dot{W}_{\text{out}}}{\dot{W}_{\text{rev}}} = \frac{\dot{m}(h_1 - h_2)}{\dot{m}[(h_1 - h_2 - T_0(s_1 - s_2)]} \tag{3.27}$$

$$\text{or} \quad \varepsilon_{\text{turbine}-1} = 1 - \frac{\dot{X}_{\text{dest}}}{\dot{X}_{\text{expended}}} = 1 - \frac{\dot{X}_{\text{dest}}}{\dot{X}_1 - \dot{X}_2} = 1 - \frac{\dot{W}_{\text{rev}} - \dot{W}_{\text{out}}}{\dot{X}_1 - \dot{X}_2} \tag{3.28}$$

The exergy efficiency definition based on the "exergy out/exergy in" approach is

$$\varepsilon_{\text{turbine}-2} = \frac{\dot{X}_{\text{out}}}{\dot{X}_{\text{in}}} = \frac{\dot{W}_{\text{out}} + \dot{X}_2}{\dot{X}_1} \neq \frac{\dot{W}_{\text{out}}}{\dot{W}_{\text{rev}}} \tag{3.29}$$

A third definition only assumes power output as the product and inlet exergy as the input:

$$\varepsilon_{\text{turbine}-3} = \frac{\dot{W}_{\text{out}}}{\dot{X}_1} = 1 - \frac{\dot{X}_{\text{dest}}}{\dot{X}_1} = 1 - \frac{\dot{W}_{\text{rev}} - \dot{W}_{\text{out}}}{\dot{X}_1} \tag{3.30}$$

Note that the first definition [(3.27)] is consistent with the general definition for the second law efficiency of work-producing devices (the ratio of actual work to reversible work), but the second and third definitions (3.29) and (3.30) are not. Also, the first definition satisfies both bounding conditions for the second law efficiency: it is 100% when actual work equals reversible work, and 0% when actual work is zero (and thus the entire expended exergy is destroyed).

It should be noted that isentropic efficiency and second law efficiency are different definitions. In isentropic efficiency, an ideal isentropic process between the actual initial state and an assumed hypothetical exit state is used whereas in exergy efficiency, an ideal reversible process between the actual inlet state and actual exit state is used. Consequently, close but different values for isentropic and exergy efficiencies are obtained. Some consequences of isentropic efficiency versus exergy efficiency for an adiabatic turbine are discussed in [14].

Example 3.2 Consider an adiabatic steam turbine with the following inlet and exit states: $P_1 = 10{,}000$ kPa, $T_1 = 500°\text{C}$, $P_2 = 10$ kPa, and $x_2 = 0.95$. Taking the dead-state temperature of steam as saturated liquid at 25°C, determine the isentropic efficiency and exergy efficiency based on different approaches.

Solution The various efficiencies are determined from (3.26, 3.27, 3.29), and (3.30) to be

$$\eta_{\text{isen, turbine}} = 0.742, \ \varepsilon_{\text{turbine}-1} = 0.812, \ \varepsilon_{\text{turbine}-2} = 0.831, \ \varepsilon_{\text{turbine}-3} = 0.729$$

That is, the second law efficiency is 72.9% based on (3.29) and it is 81.2% based on (3.27). In (3.29) and (3.30), the exergy of the steam at the turbine exit is part of the exergy destroyed by the turbine. However, the turbine should not be held responsible for the exergy it did not destroy as part of the processes associated

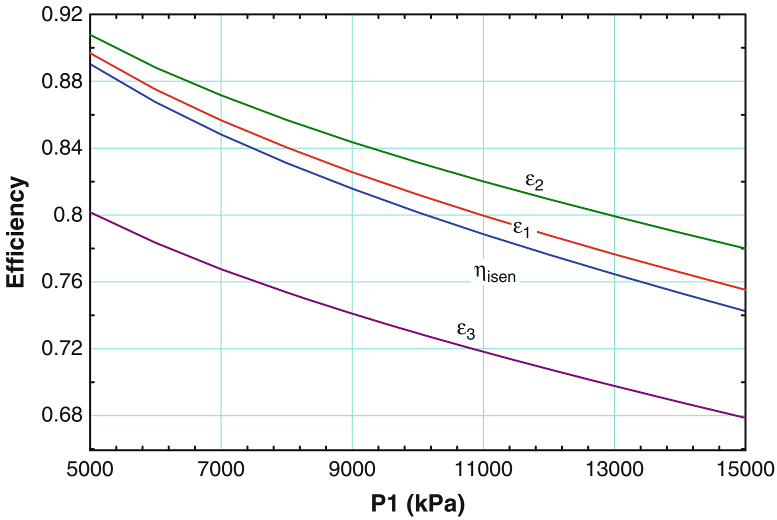


Fig. 3.4 Effect of turbine inlet pressure on the isentropic and second law efficiencies

with power production. With the first definition, the difference between the exergies of the inlet and exit steams is used for the exergy expended in the system.

The effect of turbine inlet pressure on isentropic efficiency [(3.26)] and three forms of exergy efficiencies [(3.27, 3.29, 3.30)] is investigated (Fig. 3.4). The efficiencies based on four definitions are considerably different. However, isentropic efficiency and second law efficiency by (3.27) are more appropriate forms. Interestingly, values of these two efficiencies are close to each other.

3.3.2 Compressor

A compressor is used to increase the pressure of a gas. A power input is needed for this compression process. The performance of an adiabatic compressor is usually expressed by isentropic (adiabatic) efficiency.

Consider an adiabatic compressor with inlet state 1 and an exit state 2. The power input to this compressor would be minimum if the gas were compressed reversibly and adiabatically (i.e., isentropically) between the given initial state and given exit pressure. The isentropic efficiency is then the ratio of the isentropic power to the actual power:

$$\eta_{isen,\ \text{compressor}} = \frac{\dot{W}_{\text{isentropic}}}{\dot{W}_{\text{actual}}} = \frac{\dot{m}(h_{2s} - h_1)}{\dot{m}(h_2 - h_1)} \tag{3.31}$$

where $\dot{m}$ is the mass flow rate of the gas and h_s is the enthalpy of the fluid at the compressor outlet if the process were isentropic. This enthalpy may be obtained from exit pressure and exit entropy (equal to inlet entropy). Kinetic and potential

energy changes are neglected. If the gas may be modeled as an ideal gas with constant specific heats, the isentropic power input is determined from

$$\dot{W}_{\text{isentropic}} = \dot{m}\frac{kR(T_2 - T_1)}{k-1} = \dot{m}\frac{kRT_1}{k-1}\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right] \tag{3.32}$$

where k is the specific heat ratio ($k = c_p/c_v$). Its value is 1.4 for air at room temperature.

The gas is sometimes cooled as being compressed in a nonadiabatic compressor to reduce power input. This is because the power input is proportional to the specific volume of the gas and cooling the gas decreases its specific volume. The isentropic efficiency cannot be used in such nonadiabatic compressors. Instead, an isothermal efficiency may be defined as

$$\eta_{isothermal,\,\text{compressor}} = \frac{\dot{W}_{\text{isothermal}}}{\dot{W}_{\text{actual}}} \tag{3.33}$$

where the power input for the reversible isothermal case is given for an ideal gas with constant specific heats to be

$$\dot{W}_{\text{isothermal}} = \dot{m}RT_1 \ln\left(\frac{P_2}{P_1}\right) \tag{3.34}$$

Here, R is the gas constant, T is the inlet temperature of the gas, and P_1 and P_2 are the pressures at the inlet and exit of the compressor, respectively. In some cases, a reversible polytropic process may be used as the ideal process for the actual compression. Then, a polytropic efficiency may be defined as

$$\eta_{polytropic,\,\text{compressor}} = \frac{\dot{W}_{\text{polytropic}}}{\dot{W}_{\text{actual}}} \tag{3.35}$$

where the power input for the reversible polytropic process is given for an ideal gas with constant specific heats to be

$$\dot{W}_{\text{polytropic}} = \dot{m}\frac{nR(T_2 - T_1)}{n-1} = \dot{m}\frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right] \tag{3.36}$$

Here, n is the polytropic exponent and its value changes between 1 and specific heat ratio k.

The exergy efficiency of a compressor may be determined from an "exergy recovered/exergy expended" approach. Here the resource is shaft work input (which is exergy expended by the compressor), and exergy recovered is the exergy supplied to the working fluid of the compressor, which is the increase in the exergy

of fluid as it passes through the compressor. Again taking state 1 as the inlet and state 2 as the outlet, the exergy efficiency is expressed as

$$\varepsilon_{\text{compressor}} = \frac{\dot{X}_{\text{recovered}}}{\dot{X}_{\text{expended}}} = \frac{\dot{X}_2 - \dot{X}_1}{\dot{W}_{\text{in}}} = \frac{\dot{W}_{\text{rev}}}{\dot{W}_{\text{in}}} \tag{3.37}$$

or

$$\varepsilon_{\text{compressor}} = 1 - \frac{\dot{X}_{\text{dest}}}{\dot{X}_{\text{consumed}}} = 1 - \frac{\dot{X}_{\text{dest}}}{\dot{W}_{\text{in}}} = 1 - \frac{\dot{W}_{\text{in}} - \dot{W}_{\text{rev}}}{\dot{W}_{\text{in}}} \tag{3.38}$$

The old definition based on the "exergy out/exergy in" approach is

$$\varepsilon_{\text{compressor}} = \frac{\dot{X}_{\text{out}}}{\dot{X}_{\text{in}}} = \frac{\dot{X}_2}{\dot{X}_1 + \dot{W}_{\text{in}}} \neq \frac{\dot{W}_{\text{rev}}}{\dot{W}_{\text{in}}} \tag{3.39}$$

Note that the definition of (3.37) is consistent with the general definition for the second law efficiency of work-consuming devices (the ratio of reversible work to actual work), but the definition of (3.39) is not. Also, the definition in (3.37) satisfies both bounding conditions for the second law efficiency: It is 100% when the exergy increase of the working fluid equals actual work input, and 0% when the fluid experiences no increase in exergy as it passes through the compressor (and thus the entire expended exergy is destroyed).

3.3.3 Pump

A pump is used to increase the pressure of a liquid. A power input is needed for this process. The liquid may be considered to be an incompressible fluid and the power input for the isentropic case may be determined from specific volume and pressure data. When the changes in potential and kinetic energies of a liquid are negligible, the isentropic efficiency of a pump is defined as

$$\eta_{isen,\,\text{pump}} = \frac{\dot{W}_{\text{isentropic}}}{\dot{W}_{\text{actual}}} = \frac{\dot{m}v(P_2 - P_1)}{\dot{m}(h_2 - h_1)} \tag{3.40}$$

where v is the specific volume of the liquid, and it is usually taken at the pump inlet.

One can also use (3.31) to determine isentropic efficiency of a pump. However, because the temperature change across a pump is small, it is difficult to get an accurate measurement of temperatures, and the corresponding enthalpy values are not dependable. For this reason, efficiency of a pump should be determined from the measurements of specific volume, pressures, and the actual power input. If the

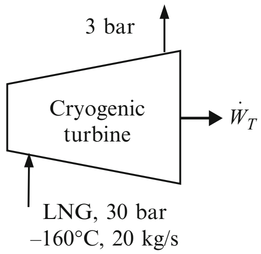


Fig. 3.5 The cryogenic turbine considered in Example 3.3

enthalpy value at the pump exit h_2 is needed, it is usually determined from a knowledge of the pump isentropic efficiency or from the measurement of actual power input.

The exergy efficiency of a pump can be determined from (3.37) as the reversible power divided by the actual power where

$$\varepsilon_{\text{pump}} = \frac{\dot{X}_{\text{recovered}}}{\dot{X}_{\text{expended}}} = \frac{\dot{X}_2 - \dot{X}_1}{\dot{W}_{\text{in}}} = \frac{\dot{W}_{\text{rev}}}{\dot{W}_{\text{in}}} = \frac{\dot{m}[h_2 - h_2 - T_o(s_2 - s_1)]}{\dot{W}_{\text{in}}} \tag{3.41}$$

We note again that it is difficult to get dependable values for enthalpy and entropy due to the small temperature change across the pump. Therefore, it is reasonable to assume that the reversible power input will be approximately equal to the isentropic power input. Then, the definitions for the exergy efficiency and isentropic efficiency become equal.

The operation of a hydraulic turbine is similar to that of a pump. Then the efficiency of a hydraulic turbine may be expressed by modifying (3.40) as

$$\eta_{isen,\ \text{hydraulic turbine}} = \frac{\dot{W}_{\text{actual}}}{\dot{W}_{\text{isentropic}}} = \frac{\dot{W}_{\text{actual}}}{\dot{m}v(P_1 - P_2)} \tag{3.42}$$

This is also called hydraulic efficiency, and it may be considered as an isentropic efficiency definition for a hydraulic turbine.

Example 3.3 Consider an adiabatic cryogenic turbine used in natural gas liquefaction plants (Fig. 3.5). Liquefied natural gas (LNG) enters a cryogenic turbine at 30 bar and $-160°C$ at a rate of 20 kg/s and leaves at 3 bar. If 115 kW power is produced by the turbine, determine the efficiency of the turbine. Take the density of LNG to be 423.8 kg/m^3.

Solution The maximum possible power that can be obtained from this turbine for the given inlet and exit pressures can be determined from

$$\dot{W}_{\text{isentropic}} = \frac{\dot{m}}{\rho}(P_1 - P_2) = \frac{20\,\text{kg/s}}{423.8\,\text{kg/m}^3}(3000 - 300)\text{kPa} = 127.4\,\text{kW}$$

Given the actual power, the efficiency of this cryogenic turbine becomes

$$\eta = \frac{\dot{W}_{\text{actual}}}{\dot{W}_{\text{isentropic}}} = \frac{115\,\text{kW}}{127.4\,\text{kW}} = 0.903 \text{ or } 90.3\%$$

3.3.4 Nozzle

A nozzle is essentially an adiabatic device because of the negligible heat transfer; it is used to accelerate a fluid. Therefore, the isentropic (i.e., reversible and adiabatic) process serves as a suitable model for nozzles. The *isentropic efficiency of a nozzle* is defined as the ratio of the actual kinetic energy of the fluid at the nozzle exit to the kinetic energy value at the exit of an isentropic nozzle for the same inlet state and exit pressure. That is,

$$\eta_{isen,\text{ nozzle}} = \frac{\text{KE}_{\text{exit, actual}}}{\text{KE}_{\text{exit, isentropic}}} = \frac{V_2^2}{V_{2s}^2} \tag{3.43}$$

When the inlet velocity is negligible, the isentropic efficiency of the nozzle can be expressed in terms of enthalpies:

$$\eta_{isen,\text{ nozzle}} = \frac{h_1 - h_2}{h_1 - h_{2s}} \tag{3.44}$$

A nozzle is built to convert the enthalpy of a fluid to kinetic energy, just like a turbine being built to convert the enthalpy of a fluid to shaft work. In a nozzle, the exergy recovered is the increase in the kinetic energy of the fluid and exergy expended is the exergy decrease of the fluid stream (but without taking into consideration the exit kinetic energy of the fluid stream, which corresponds to shaft work in a turbine). Then the exergy efficiency of a nozzle may be defined using an "exergy recovered/exergy expended" approach as

$$\varepsilon_{\text{nozzle}} = \frac{\dot{X}_{\text{recovered}}}{\dot{X}_{\text{expended}}} \tag{3.45}$$

and in a more explicit manner it becomes

$$\begin{aligned}\varepsilon_{\text{nozzle}} &= \frac{\dot{X}_{\text{recovered}}}{\dot{X}_{\text{expended}}} = \frac{V_2^2/2 - V_1^2/2}{h_1 - h_2 - T_0(s_1 - s_2) + V_1^2/2} \\ &\cong \frac{V_2^2/2}{h_1 - h_2 - T_0(s_1 - s_2)}\end{aligned} \tag{3.46}$$

Note that expended exergy corresponds to the exit kinetic energy in a reversible process. When the nozzle is reversible and adiabatic, the exergy efficiency becomes 100%, as expected. For adiabatic nozzles, isentropic and exergy efficiencies are identical. For nonadiabatic nozzles, the denominator of (3.46) can be modified to include the exergy of the heat transferred.

3.3.5 Throttling Valve

A throttling valve is used to decrease the pressure of a fluid in a constant-enthalpy process. Because the energy of the mass at the inlet and exit of the valve are the same, it is not appropriate to define an efficiency based on the first law of thermodynamics. However, the adiabatic expansion process in a throttling valve is a highly irreversible process, and thus a second law efficiency may be defined.

Here the resource is the high pressure and temperature fluid, and exergy expended is the exergy supplied by the fluid, which is the decrease of the exergy of the fluid as it passes through the valve. There is no exergy recovered in any form, and thus exergy recovered is zero. Therefore:

$$\varepsilon_{\text{exp valve}} = \frac{\dot{X}_{\text{recovered}}}{\dot{X}_{\text{expended}}} = \frac{0}{\dot{X}_1 - \dot{X}_2} = 0 \tag{3.47}$$

$$\text{or} \quad \varepsilon_{\text{exp valve}} = 1 - \frac{\dot{X}_{\text{dest}}}{\dot{X}_{\text{expended}}} = 1 - \frac{\dot{X}_1 - \dot{X}_2}{\dot{X}_1 - \dot{X}_2} = 0 \tag{3.48}$$

This indicates that the second law efficiency of a throttling valve is always zero, which is expected because a throttling valve makes no use of the fluid exergy it expends. The commonly used definition for the exergy efficiency of a throttling valve is:

$$\varepsilon_{\text{exp valve}} = \frac{\dot{X}_{\text{out}}}{\dot{X}_{\text{in}}} = \frac{\dot{X}_2}{\dot{X}_1} \tag{3.49}$$

$$\text{or} \quad \varepsilon_{\text{exp valve}} = 1 - \frac{\dot{X}_{\text{dest}}}{\dot{X}_{\text{in}}} = 1 - \frac{\dot{X}_1 - \dot{X}_2}{\dot{X}_1} = \frac{\dot{X}_2}{\dot{X}_1} \tag{3.50}$$

Note that the current definition gives misleading results. A throttling valve that reduces the exergy rate of a fluid from 100 to 90 kW, for example, would have a second law efficiency of 90% according to the current definition, which is impressive. But this is unrealistic for such a wasteful device.

By similar reasoning, we can infer that the second law efficiency of electric transmission lines is zero inasmuch as the dissipated electric power is converted to heat which is rejected to the environment at environment temperature. Also, the

second law efficiency of steam pipes losing heat to the environment is zero. Here the exergy of lost heat is zero because it is at the environment temperature, and no attempt is made to recover its exergy when the lost heat is at the outer surface temperature of the pipe.

3.3.6 Heat Exchanger

In a heat exchanger, two fluid streams exchange heat without mixing. When it comes to defining a second law or exergy efficiency for a heat exchanger, general disagreement is the rule rather than the exception. This is because there are several ways to define the exergy source. One way is to consider the hot fluid stream as the source of exergy and to disregard the cold fluid stream as a potential exergy source. Another way is to consider the exergy content of both fluid streams as the exergy source. Things are complicated even further when the cold fluid stream is below the environment temperature and the hot fluid stream is above so that the exergy of both fluid streams decreases during heat exchange.

When we deal with fluid streams above the environment temperature T_0, which is most often the case in practice, the cold fluid stream experiences an increase in its exergy content and none of the exergy of the incoming cold stream is expended. Therefore, we believe only the hot fluid stream should be considered as the exergy source in such cases, and the exergy increase of the cold fluid stream represents the exergy recovered. Then the exergy efficiency of a heat exchanger can be defined as

$$\varepsilon_{\text{heat exchanger}} = \frac{\dot{X}_{\text{recovered}}}{\dot{X}_{\text{expended}}} = \frac{\left(\dot{X}_{\text{out}} - \dot{X}_{\text{in}}\right)_{\text{cold}}}{\left(\dot{X}_{\text{in}} - \dot{X}_{\text{out}}\right)_{\text{hot}}} = 1 - \frac{\dot{X}_{\text{dest}}}{\left(\dot{X}_{\text{in}} - \dot{X}_{\text{out}}\right)_{\text{hot}}} \tag{3.51}$$

If the cold fluid is heated from T_1 to T_2 and the hot fluid is cooled from T_3 to T_4 at specified pressures, this expression may be written as

$$\varepsilon_{\text{heat exchanger}} = \frac{\left(\dot{X}_{\text{out}} - \dot{X}_{\text{in}}\right)_{\text{cold}}}{\left(\dot{X}_{\text{in}} - \dot{X}_{\text{out}}\right)_{\text{hot}}} = \frac{\dot{m}_{\text{cold}}[h_2 - h_1 - T_0(s_2 - s_1)]}{\dot{m}_{\text{hot}}[h_3 - h_4 - T_0(s_3 - s_4)]} \tag{3.52}$$

This relation will result in an exergy efficiency of 100% for a perfect counterflow heat exchanger where two identical fluids enter the heat exchanger at the same flow rates and the cold fluid is heated to the inlet temperature of the hot fluid while the hot fluid is cooled to the inlet temperature of the cold fluid. The exergy efficiency will be 0% for a heat exchanger that loses heat to its surroundings without transferring any heat to the cold fluid stream if the immediate surroundings of the heat exchanger are also taken as part of the system. Equation 3.51 is also valid if the heat exchanger is losing heat to the environment at temperature T_0 provided that the temperature gradient region between the heat exchanger and the environment is

included in the analysis. If the heat exchanger is losing heat at a rate of $\dot{Q}_{\text{loss}}$ to a medium at T_R, the recovered exergy will also include the exergy stored in the medium at T_R as a result of this heat transfer. Then the exergy efficiency relation becomes

$$\varepsilon_{\text{heat exchanger}} = \frac{\dot{X}_{\text{recovered}}}{\dot{X}_{\text{expended}}} = \frac{\left(\dot{X}_{\text{out}} - \dot{X}_{\text{in}}\right)_{\text{cold}} + \dot{Q}_{\text{loss}}(1 - T_0/T_R)}{\left(\dot{X}_{\text{in}} - \dot{X}_{\text{out}}\right)_{\text{hot}}}$$
$$= 1 - \frac{\dot{X}_{\text{dest}}}{\left(\dot{X}_{\text{in}} - \dot{X}_{\text{out}}\right)_{\text{hot}}} \tag{3.53}$$

Special Case 1: Let us consider a heat exchanger where the hot fluid is cooled to a temperature above the environment temperature T_0, and the cold fluid remains below the environment temperature T_0 between the inlet and exit as it is heated. In this case, the exergies of both the hot and cold fluids will decrease and none of this exergy will be recovered. Therefore, in this case, it is more appropriate to take the "expended exergy" as the sum of the exergy decrease of the cold and hot fluids. This sum will be equal to the exergy destruction. As a result, the exergy efficiency in this case will be zero. That is,

$$\varepsilon_{\text{heatexchanger}} = \frac{\dot{X}_{\text{recovered}}}{\dot{X}_{\text{expended}}} = \frac{0}{\left(\dot{X}_{\text{in}} - \dot{X}_{\text{out}}\right)_{\text{hot}} + \left(\dot{X}_{\text{in}} - \dot{X}_{\text{out}}\right)_{\text{cold}}} = 0 \tag{3.54}$$

or

$$\varepsilon_{\text{heat exchanger}} = 1 - \frac{\dot{X}_{\text{dest}}}{\dot{X}_{\text{expended}}} = 1 - \frac{\left(\dot{X}_{\text{in}} - \dot{X}_{\text{out}}\right)_{\text{hot}} + \left(\dot{X}_{\text{in}} - \dot{X}_{\text{out}}\right)_{\text{cold}}}{\left(\dot{X}_{\text{in}} - \dot{X}_{\text{out}}\right)_{\text{hot}} + \left(\dot{X}_{\text{in}} - \dot{X}_{\text{out}}\right)_{\text{cold}}} = 0 \tag{3.55}$$

Special Case 2: What happens if the cold fluid is heated from a temperature T_1, which is below the environment temperature T_0 to a temperature T_2, which is greater than T_0 and the hot fluid is cooled from a temperature T_3, which is greater than T_0 to a temperature T_4, which is lower than T_0. In this case, the entire exergy contents of both the hot and cold fluid streams are expended, and thus the expended exergy is the sum of the initial exergies of the hot and cold fluid streams. But the hot fluid stream recovers part of this exergy as it is cooled to the subenvironment temperature and the cold fluid recovers part of its exergy as it is heated to the above environment temperature. Therefore, the total recovered exergy is the sum of the final exergies of the hot and cold fluid streams. Then the exergy efficiency in this case can be expressed as

$$\varepsilon_{\text{heatexchanger}} = \frac{\dot{X}_{\text{recovered}}}{\dot{X}_{\text{expended}}} = \frac{\dot{X}_{\text{out, cold}} + \dot{X}_{\text{out, hot}}}{\dot{X}_{\text{in, cold}} + \dot{X}_{\text{in, hot}}} = \frac{\dot{X}_2 + \dot{X}_4}{\dot{X}_1 + \dot{X}_3} \tag{3.56}$$

where

$$\dot{X}_1 = \dot{m}_{\text{cold}}[h_1 - h_0 - T_0(s_1 - s_0)] \quad \dot{X}_2 = \dot{m}_{\text{cold}}[h_2 - h_0 - T_0(s_2 - s_0)]$$
$$\dot{X}_3 = \dot{m}_{\text{hot}}[h_3 - h_0 - T_0(s_3 - s_0)] \quad \dot{X}_4 = \dot{m}_{\text{cold}}[h_4 - h_0 - T_0(s_4 - s_0)] \tag{3.57}$$

These relations are consistent with our general criteria that in the absence of any irreversibilities the exergy efficiency should be 100%. Indeed, the exergy efficiency for a perfect counterflow heat exchanger will be 100% if two identical fluid streams, one at $T_1 < T_0$ and the other at $T_2 > T_{0,}$ enter the heat exchanger with identical mass flow rates because the cold fluid will be heated to T_2 and the hot fluid will be cooled to T_1.

To generalize, we can say that in the second law analysis of heat exchangers, all fluid streams that experience a decrease in their exergy content are to be considered in the evaluation of the expended exergy. Likewise, all fluid streams that experience an increase in their exergy content are to be considered in the evaluation of the recovered exergy. A fluid stream that crosses the dead-state is to be considered in the evaluation of both the expended exergy and recovered exergy. For such a fluid stream the expended exergy is the exergy at the inlet state, and the recovered exergy is the exergy at the exit state.

Effectiveness of a Heat Exchanger: The performance of heat exchangers is usually expressed by their effectiveness. It is then defined as

$$\eta_{\text{eff, heat exchanger}} = \frac{\dot{Q}_{\text{actual}}}{\dot{Q}_{\max}} = \frac{\dot{Q}_{\text{actual}}}{\left(\dot{m}c_p\right)_{\min}\left(T_{\text{hot,in}} - T_{\text{cold,in}}\right)}$$
$$= \frac{\left(\dot{m}c_p\Delta T\right)_{\text{cold or hot}}}{\left(\dot{m}c_p\right)_{\min}\left(T_{\text{hot, in}} - T_{\text{cold, in}}\right)} \tag{3.58}$$

or using enthalpies

$$\eta_{\text{eff, heat exchanger}} = \frac{\dot{Q}_{\text{actual}}}{\dot{Q}_{\max}} = \frac{\left(\dot{m}\Delta h\right)_{\text{cold or hot}}}{\dot{m}_{\min}\left(h_{\text{hot, in}} - h_{\text{cold, in}}\right)} \tag{3.59}$$

where $(\dot{m}c_p)_{\min}$ is the smaller of the heat capacity rate between hot and cold fluids, and $\dot{m}_{\min}$ is the smaller mass flow rate. The effectiveness of a heat exchanger is 100% when it transfers the maximum amount of heat transfer, and is 0% when it transfers no heat from the hot to the cold fluid.

Example 3.4 Calculate the exergy efficiency and effectiveness of a heat exchanger with the following data.

$$T_{1,\text{ hot}} = 60°\text{C},\ T_{2,\text{ hot}} = 35°C,\ \dot{m}_{\text{hot}} = 1\,\text{kg/s},\ c_{p,\text{ hot}} = 4.18\,\text{kJ/kg}\cdot°\text{C}$$

$$T_{3,\text{ cold}} = 5°\text{C},\ T_{4,\text{ cold}} = 20°\text{C},\ c_{p,\text{ cold}} = 4.18\,\text{kJ/kg}\cdot°\text{C},\ T_0 = 25°\text{C}$$

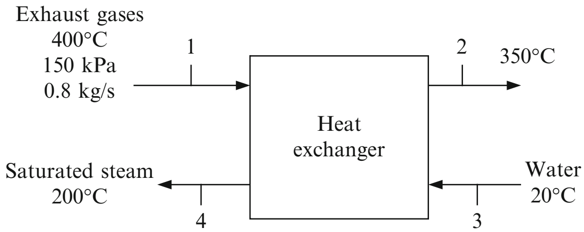


Fig. 3.6 A cross-flow heat exchanger used to produce steam

Solution This heat exchanger operates as explained in Special Case 1. The exergy decreases of hot and cold fluids are determined to be 7.3 and 4.6 kW, respectively. The sum of these is 11.9 kW, and it is equal to the expended exergy. None of this expended exergy is recovered, therefore this is also equal to the exergy destruction, and the exergy efficiency is zero. The effectiveness is calculated from (3.58) as 0.456 or 45.6%.

Example 3.5 Calculate the exergy efficiency and effectiveness of a heat exchanger with the following data.

$$T_{1,\,\text{hot}} = 60°\text{C},\ T_{2,\,\text{hot}} = 15°\text{C},\ \dot{m}_{\text{hot}} = 1\,\text{kg/s},\ c_{p,\,\text{hot}} = 4.18\,\text{kJ/kg}\cdot°\text{C}$$

$$T_{3,\,\text{cold}} = 10°\text{C},\ T_{4,\,\text{cold}} = 35°\text{C},\ c_{p,\,\text{cold}} = 4.18\,\text{kJ/kg}\cdot°\ \text{C},\ T_0 = 25°\text{C}$$

Solution This heat exchanger operates as explained in Special Case 2. Then, the exergy efficiency can be calculated using (3.56) to be 0.179 or 17.9%. The effectiveness is calculated from (3.58) as 0.90 or 90.0%.

Example 3.6 Hot exhaust gases leaving an internal combustion engine at 400°C and 150 kPa at a rate of 0.8 kg/s are to be used to produce saturated steam at 200°C in an insulated heat exchanger (Fig. 3.6). Water enters the heat exchanger at the ambient temperature of 20°C, and the exhaust gases leave the heat exchanger at 350°C. Determine the rate of steam production, the rate of exergy destruction in the heat exchanger, and the exergy efficiency of the heat exchanger.

Solution We denote the inlet and exit states of exhaust gases by (1) and (2) and that of the water by (3) and (4). The properties of water are obtained from the steam tables to be

$$T_3 = 20°\text{C},\ \text{liquid} \longrightarrow h_3 = 83.91\,\text{kJ/kg}, s_3 = 0.29649\,\text{kJ/kg.K}$$
$$T_4 = 200°\text{C},\ \text{saturated vapor} \longrightarrow h_4 = 2792.0\,\text{kJ/kg}, s_4 = 6.4302\,\text{kJ/kg.K}$$

An energy balance on the heat exchanger gives the rate of steam production:

$$\begin{aligned}
\dot{m}_a h_1 + \dot{m}_w h_3 &= \dot{m}_a h_2 + \dot{m}_w h_4 \\
\dot{m}_a c_p (T_1 - T_2) &= \dot{m}_w (h_4 - h_3) \\
(0.8\,\text{kg/s})(1.063\,\text{kJ/kg°C})(400 - 350)°\text{C} &= \dot{m}_w (2792.0 - 83.91)\text{kJ/kg} \\
\dot{m}_w &= 0.01570\,\text{kg/s}
\end{aligned}$$

The specific exergy changes of air and water streams as they flow in the heat exchanger are

$$\begin{aligned}\Delta\psi_a &= c_p(T_2 - T_1) - T_0(s_2 - s_1)\\ &= (1.063\,\text{kJ/kg.°C})(350 - 400)\text{°C} - (20 + 273\text{K})(-0.08206\,\text{kJ/kg.K})\\ &= -29.106\,\text{kJ/kg}\end{aligned}$$

$$\begin{aligned}\Delta\psi_w &= (h_4 - h_3) - T_0(s_4 - s_3)\\ &= (2792.0 - 83.91)\text{kJ/kg} - (20 + 273\,\text{K})(6.4302 - 0.29649)\,\text{kJ/kg.K}\\ &= 910.913\,\text{kJ/kg}\end{aligned}$$

The exergy destruction is determined from an exergy balance as

$$(\dot{m}_a\psi_1 + \dot{m}_w\psi_3) - (\dot{m}_a\psi_3 + \dot{m}_w\psi_4) - \dot{X}_{\text{destroyed}} = 0$$

Rearranging and substituting,

$$\begin{aligned}\dot{X}_{\text{destroyed}} &= \dot{m}_a\Delta\psi_a + \dot{m}_w\Delta\psi_w\\ &= (0.8\,\text{kg/s})(-29.106\,\text{kJ/kg}) + (0.01570\,\text{kg/s})(910.913)\text{kJ/kg}\\ &= 8.98\,\text{kW}\end{aligned}$$

The exergy efficiency for a heat exchanger may be defined as the exergy increase of the cold fluid divided by the exergy decrease of the hot fluid. That is,

$$\varepsilon = \frac{\dot{m}_w\Delta\psi_w}{-\dot{m}_a\Delta\psi_a} = \frac{(0.01570\,\text{kg/s})(910.913\,\text{kJ/kg})}{-(0.8\,\text{kg/s})(-29.106\,\text{kJ/kg})} = 0.614 \text{ or } 61.4\%$$

The energy efficiency of this heat exchange process is 100% because it is assumed that all the energy given up by the exhaust gases are picked up by the water. The process is perfect from an energetic point of view whereas it is far from ideal from an exergetic perspective. The exergy destruction during this process is due to heat transfer across a finite temperature difference. This is illustrated in Fig. 3.7, which shows the exergy efficiency of the heat exchanger as a function of the exhaust gas inlet temperature for the same temperature drop of 50°C for the exhaust gases. As the average temperature difference between the exhaust gases and the water increases, the exergy efficiency decreases. In another words, the larger the temperature difference is, the larger the exergy destruction.

3.3.7 Mixing Chamber

Two fluid streams mix to produce a third fluid stream in a mixing chamber. When both incoming fluid streams are above the environment temperature, the exergy resource is the hot fluid, and the exergy expended is the exergy decrease of the hot fluid. The exergy recovered is the exergy increase of the cold fluid. Taking state 1 as the hot fluid inlet, state 2 as the cold fluid inlet, and state 3 as the common state of the mixture,

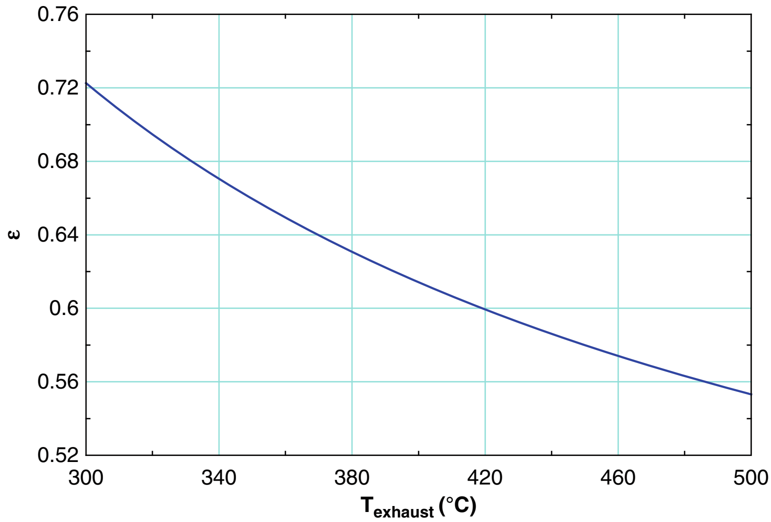


Fig. 3.7 The exergy efficiency of a heat exchanger as a function of exhaust gas inlet temperature

$$\begin{aligned}\varepsilon_{\text{mixing chamber}} &= \frac{\dot{X}_{\text{recovered}}}{\dot{X}_{\text{expended}}} = \frac{\left(\dot{X}_{\text{out}} - \dot{X}_{\text{in}}\right)_{\text{cold}}}{\left(\dot{X}_{\text{in}} - \dot{X}_{\text{out}}\right)_{\text{hot}}} = \frac{\dot{X}_3 - \dot{X}_2}{\dot{X}_1 - \dot{X}_3} \\ &= \frac{\dot{m}_{\text{cold}}[h_3 - h_2 - T_0(s_3 - s_2)]}{\dot{m}_{\text{hot}}[h_1 - h_3 - T_0(s_1 - s_3)]} = 1 - \frac{\dot{X}_{\text{dest}}}{\left(\dot{X}_{\text{in}} - \dot{X}_{\text{out}}\right)_{\text{hot}}}\end{aligned} \tag{3.60}$$

Noting that $\dot{m}_{\text{cold}}(h_3 - h_2) = \dot{m}_{\text{hot}}(h_1 - h_3)$, manipulating the last equality gives $\dot{X}_{\text{dest}} = T_0[\dot{m}_{\text{hot}}(s_1 - s_3) + \dot{m}_{\text{cold}}(s_2 - s_3)]$, which provides a check.

If the mixing chamber is losing heat at a rate of $\dot{Q}_{\text{loss}}$ to a medium at T_R, the recovered exergy will also include the exergy associated with heat transfer,

$$\varepsilon_{\text{mixing chamber}} = \frac{\dot{m}_{\text{cold}}[h_3 - h_2 - T_0(s_3 - s_2)] + \dot{Q}_{\text{loss}}(1 - T_0/T_R)}{\dot{m}_{\text{hot}}[h_1 - h_3 - T_0(s_1 - s_3)]} \tag{3.61}$$

When the cold stream is below the environment temperature, arguments similar to those given above for heat exchangers can be given.

3.3.8 Electric Resistance Heating

Here the resource is the electrical energy in the grid, and the exergy expended is the exergy of electricity expended by the resistance heater. If the heater is indoors at

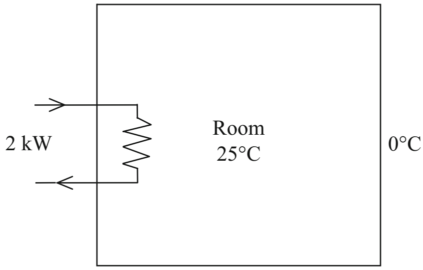


Fig. 3.8 An electric resistance heater used to heat a room

temperature T_{room} in an environment at temperature T_0, the exergy recovered is the exergy content of supplied heat to the room at room temperature:

$$\varepsilon_{\text{electric heater}} = \frac{\dot{X}_{\text{recovered}}}{\dot{X}_{\text{expended}}} = \frac{\dot{X}_{\text{heat}}}{\dot{W}_e} = \frac{\dot{Q}_e(1 - T_0/T_{\text{room}})}{\dot{W}_e} = 1 - \frac{T_0}{T_{\text{room}}} \tag{3.62}$$

Note that the second law efficiency of a resistance heater becomes zero when the heater is outdoors.

Example 3.7 An electric resistance heater with a power consumption of 2.0 kW is used to heat a room at 25°C when the outdoor temperature is 0°C (Fig. 3.8). Determine energy and exergy efficiencies and the rate of exergy destroyed for this process.

Solution For each unit of electric work consumed, the heater will supply the house with 1 unit of heat. That is, the heater has a COP of 1. Also, the energy efficiency of the heater is 100% because the energy output (heat supply to the room) and the energy input (electric work consumed by the heater) are the same. At the specified indoor and outdoor temperatures, a reversible heat pump would have a COP of

$$\text{COP}_{\text{HP, rev}} = \frac{1}{1 - T_L/T_H} = \frac{1}{1 - (273\,\text{K})/(298\,\text{K})} = 11.9$$

That is, it would supply the house with 11.9 units of heat (extracted from the cold outside air) for each unit of electric energy it consumes (Fig. 3.9). The exergy efficiency of this resistance heater is

$$\varepsilon = \frac{\text{COP}}{\text{COP}_{\text{HP, rev}}} = \frac{1}{11.9} = 0.084 \text{ or } 8.4\%$$

The minimum work requirement to the heater is determined from the COP definition for a heat pump to be

$$\dot{W}_{\text{in, min}} = \frac{\dot{Q}_{\text{supplied}}}{\text{COP}_{\text{HP, rev}}} = \frac{2\,\text{kW}}{11.9} = 0.17\,\text{kW}$$

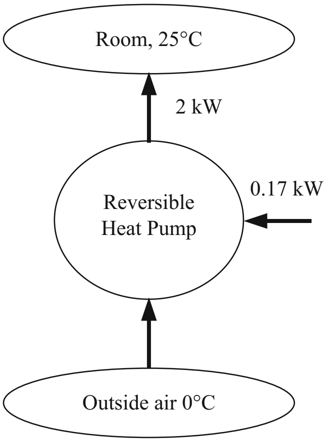


Fig. 3.9 A reversible heat pump consuming only 0.17 kW power while supplying 2 kW of heat to a room

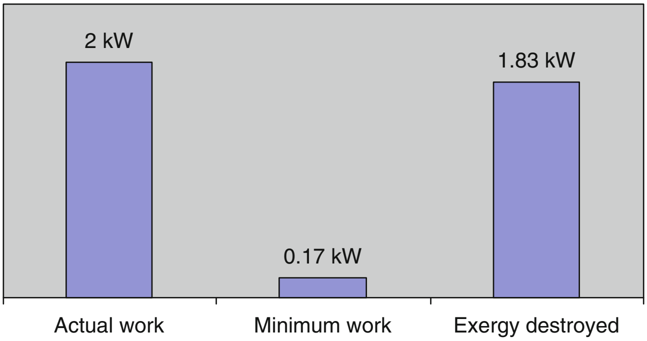


Fig. 3.10 Comparison of actual and minimum works with the exergy destroyed

That is, a reversible heat pump would consume only 0.17 kW of electrical energy to supply the room with 2 kW of heat. The exergy destroyed is the difference between the actual and minimum work input:

$$\dot{X}_{\text{destroyed}} = \dot{W}_{\text{in}} - \dot{W}_{\text{in, min}} = 2.0 - 0.17 = 1.83\,\text{kW}$$

The results of this example are illustrated in Figs. 3.10 and 3.11. The performance looks perfect with respect to energy efficiency but not so good from the point of view of exergy efficiency. About 92% of actual work input to the resistance heater is wasted during the operation of the resistance heater. There must be better methods of heating this room. Using a heat pump (preferably a ground-source one) or a natural

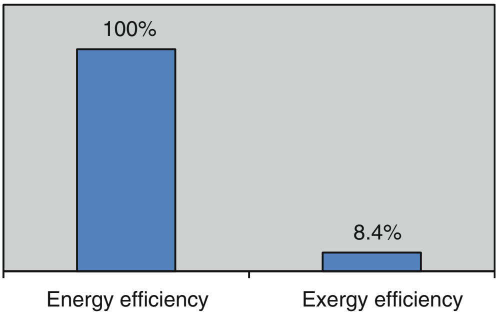


Fig. 3.11 Comparison of energy and exergy efficiencies

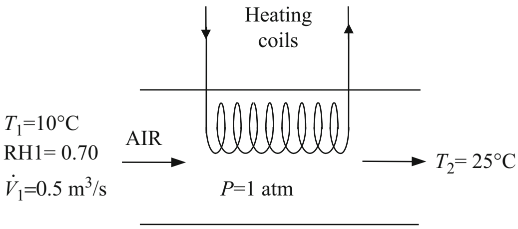


Fig. 3.12 Schematic of a simple heating process

gas furnace would involve lower exergy destruction and correspondingly greater exergy efficiencies even though the energy efficiency of a natural gas furnace is lower than that of a resistance heater.

Different heating systems may also be compared using the primary energy ratio (PER), which is the ratio of useful heat delivered to the primary energy input. Obviously, the higher the PER is, the more efficient the heating system. The PER for a heat pump is defined as PER $= \eta \times$ COP where η is the thermal efficiency with which the primary energy input is converted into work. For the resistance heater discussed in this example, the thermal efficiency η may be taken to be 0.40 if the electricity is produced from a natural gas-fueled steam power plant. Because the COP is 1, the PER becomes 0.40. A natural gas furnace with an efficiency of 0.80 (i.e., heat supplied over the heating value of the fuel) would have a PER value of 0.80. Furthermore, with a ground-source heat pump using electricity as the work input, the COP may be taken as 3 and with the same method of electricity production ($\eta = 0.40$), the PER becomes 1.2.

Example 3.8 In an air-conditioning process, air is heated by a heating coil in which hot water is flowing at an average temperature of 80°C. Using the values given in Fig. 3.12, determine the exergy destruction and the exergy efficiency for this process.

Solution The properties of air at various states (including dead-state, denoted by the subscript 0) are determined from software with built-in properties to be

$$v_1 = 0.810\ \text{m}^3/\text{kg},\ h_0 = h_1 = 25.41\,\text{kJ/kg},\ h_2 = 40.68\,\text{kJ/kg},$$
$$s_0 = s_1 = 5.701\,\text{kJ/kg}\cdot\text{K}\ s_2 = 5.754\,\text{kJ/kg}\cdot\text{K},$$
$$w_1 = w_2 = 0.00609\,\text{kg water/kg air},\ \text{RH}_2 = 0.31$$

The dead-state temperature is taken to be the same as the inlet temperature of air. The mass flow rate of air and the rate of heat input are

$$\dot{m}_a = \frac{\dot{V}_a}{v_1} = 0.617\,\text{kg/s}$$
$$\dot{Q}_{\text{in}} = \dot{m}_a(h_2 - h_1) = 9.43\,\text{kW}$$

The exergies of the air stream at the inlet and exit become

$$\dot{X}_1 = 0 \quad \text{and} \quad \dot{X}_2 = \dot{m}_a[(h_2 - h_0) - T_0(s_2 - s_0)] = 0.267\,\text{kW}$$

The rates of exergy input and the exergy destroyed are

$$\dot{X}_{\text{in}} = \dot{Q}_{\text{in}}\left(1 - \frac{T_0}{T_{\text{source}}}\right) = 1.87\,\text{kW}$$

$$\dot{X}_{\text{destroyed}} = \dot{X}_{\text{in}} - \dot{X}_{\text{out}} = 1.87 - 0.267 = 1.60\,\text{kW}$$

where the temperature at which heat is transferred to the air stream is taken as the average temperature of water flowing in the heating coils (80°C). The exergy efficiency is

$$\varepsilon = \frac{\dot{X}_{\text{out}}}{\dot{X}_{\text{in}}} = \frac{0.267\,\text{kW}}{1.87\,\text{kW}} = 0.143 \text{ or } 14.3\%$$

About 86% of exergy input is destroyed due to irreversible heat transfer in the heating section. Air-conditioning processes typically involve high rates of exergy destructions as high-temperature (i.e., high-quality) heat or high-quality electricity is used to obtain a low-quality product. The irreversibilities can be minimized using lower quality energy sources and fewer irreversible processes. For example, if heat is supplied at an average temperature of 60°C instead of 80°C, the exergy destroyed would decrease from 1.60 to 1.15 kW and the exergy efficiency would increase from 14.3% to 18.8%. The exit temperature of air also affects the exergy efficiency. For example, if air is heated to 20°C instead of 25°C, the exergy efficiency would decrease from 14.3% to 10.1%. These two examples also show that the smaller the temperature difference between the heat source and the air being heated is, the larger the exergy efficiency.